STATISTICS · DESCRIPTIVE STATISTICS

Index Numbers and Composite Indicators

How price and quantity index numbers summarise change over time, how composite indicators combine many measures into one score, and how choices of formula, weights and aggregation shape the results.

ModQu research article · 1,261 words · 12 references (APA 7th edition)

Many of the numbers that steer business and public decisions are not direct measurements at all. The consumer price index, a brand health score, a customer experience index and a national competitiveness ranking each compress many separate figures into a single number. Two related families of tools do this work. **Index numbers** summarise how a set of prices or quantities has changed relative to a reference point. **Composite indicators** combine several different measures, often on different scales, into one score for comparing units such as countries, regions, stores or brands. Both depend on choices that are easy to overlook.

Simple index numbers

The simplest index expresses a value in one period as a percentage of its value in a chosen **base period**:

$$I_t = 100 \times x_t \div x_0$$

If an average order value rises from $40 in the base year to $46 this year, the index is 115, meaning a 15% rise. Indexing removes units, so different series can share one chart. Choose a normal base period; an unusually weak base month makes every later figure look strong.

Weighted price indices

Real price indices cover many items, and the items matter to different degrees. The classic solution is to price a fixed **basket** of goods. The two oldest formulas differ only in which basket they use (Balk, 2008):

$$\text{Laspeyres: } P_L = 100 \times \Sigma\, p_t q_0 \div \Sigma\, p_0 q_0$$

$$\text{Paasche: } P_P = 100 \times \Sigma\, p_t q_t \div \Sigma\, p_0 q_t$$

Here *p* is price, *q* is quantity, 0 is the base period and *t* the current period. The **Laspeyres** index asks what the base-period basket would cost today; the **Paasche** index asks what today's basket would have cost in the base period. The **Fisher** index is the geometric mean of the two, $\sqrt{(P_L \times P_P)}$. Diewert (1976) showed that the Fisher index and a few similar formulas are "superlative": they closely approximate a true cost-of-living index while allowing buyers to substitute between goods.

A worked example

Consider a hypothetical three-item basket. In quarter 0, a household buys 100 units of item A at $4.00, 150 of item B at $2.00 and 60 of item C at $5.00, a total of $1,000. By quarter 4, prices are $6.00, $2.20 and $5.40, and because item A has become much dearer the household now buys 70 of A, 190 of B and 62 of C.

- The quarter 0 basket at quarter 4 prices costs $1,254, so the Laspeyres index is **125.4**.
- The quarter 4 basket costs $1,172.80 at current prices and $970 at base prices, so the Paasche index is 1,172.8 ÷ 970 × 100 = **120.9**.
- The Fisher index is √(125.4 × 120.9) = **123.1**.

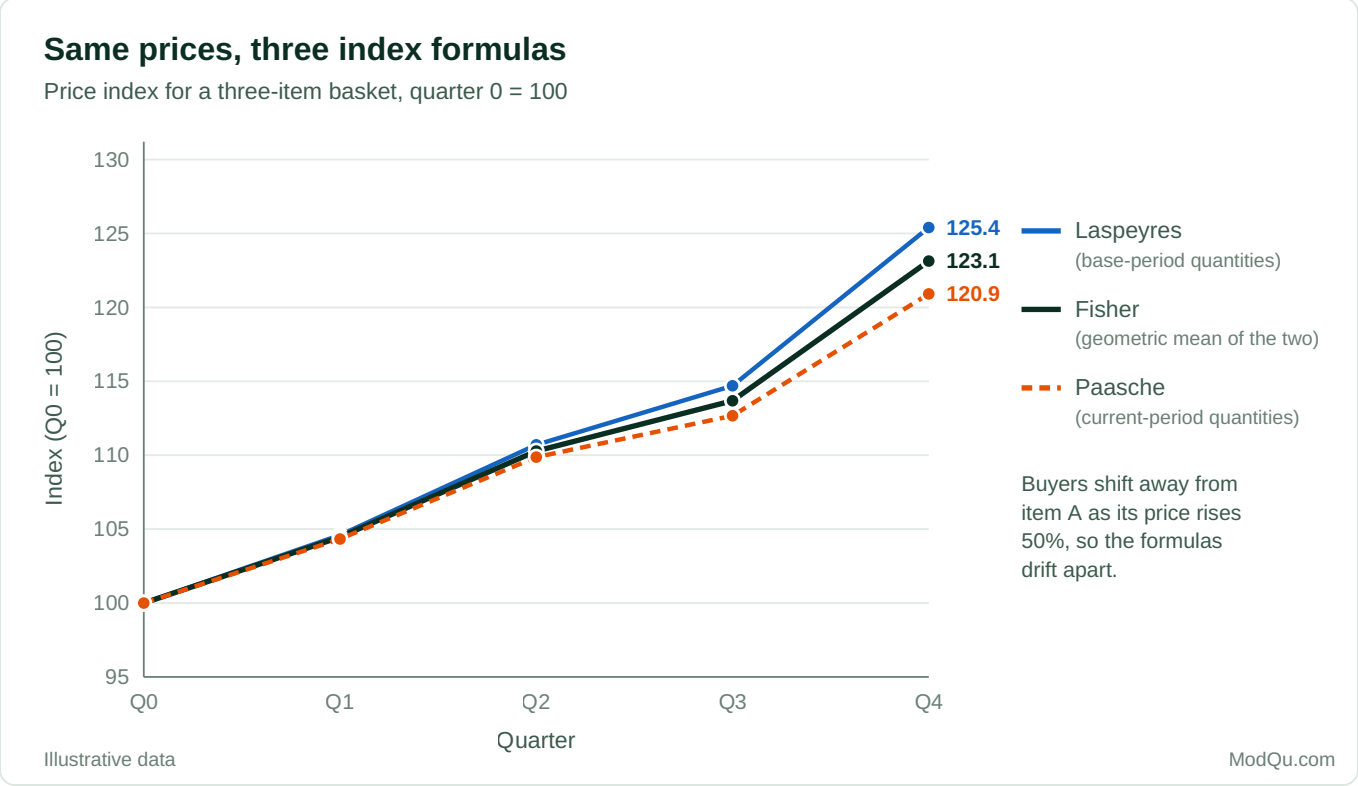


Figure 1. The same prices give different measures of inflation depending on whose basket is used. Laspeyres overstates and Paasche understates the cost-of-living change when buyers substitute. Illustrative data; chart by ModQu.

The four-point gap is **substitution bias**. Laspeyres ignores the fact that buyers moved away from item A, so it tends to overstate the rise in living costs; Paasche tends to understate it. A review of the United States consumer price index estimated its overall upward bias, from substitution, new products, quality change and outlet shifts, at about 1.1 percentage points a year (Boskin et al., 1998), which prompted many methodological changes (Diewert, 1998).

Chaining and rebasing

Over long periods, a fixed basket becomes outdated. Statistical agencies therefore **chain** indices, updating weights regularly and linking period-to-period changes together. Chaining reduces substitution bias but can introduce **chain drift** when prices and quantities bounce around, as with frequent promotions: a chained index may fail to return to its starting value even when prices and quantities return to their original levels. Ivancic et al. (2011) showed that this drift can be severe with high-frequency supermarket scanner data, a warning for any business building a price tracker from transaction records.

Composite indicators

A composite indicator combines several sub-indicators into a single score. Examples include well-being indices, innovation scoreboards and, in market research, brand equity or customer experience indices built from survey items such as awareness, satisfaction and likelihood to recommend. The *Handbook on Constructing Composite Indicators* sets out a widely used sequence of steps (Organisation for Economic Co-operation and Development [OECD] et al., 2008):

1. Define the concept and a theoretical framework.
2. Select indicators that measure it, and check their quality.
3. Treat missing values and outliers.
4. **Normalise** the indicators to a common scale.
5. **Weight** the indicators.
6. **Aggregate** them into one score.
7. Test robustness, and report the results with their components visible.

Normalisation

Indicators measured on different scales must be made comparable. Common methods include **min–max** rescaling to 0–100, **z-scores** (subtract the mean, divide by the standard deviation) and ranking. Min–max scores are easy to explain but sensitive to extreme values. The choice can change the final ranking even when the data are identical.

Weighting and aggregation

Weights may come from expert judgement, stakeholder consultation, statistical methods such as principal components, or simple equality. None is neutral (Decancq & Lugo, 2013). A common misunderstanding is that a weight equals an indicator's importance in the final score. In practice, an indicator's influence also depends on its variance and its correlation with the other indicators, so nominal weights and actual importance can differ substantially (Becker et al., 2017; Paruolo et al., 2013).

Aggregation determines how far a strength in one area can make up for a weakness in another. **Arithmetic** (additive) aggregation is fully compensatory: a brand with excellent awareness and poor satisfaction can score the same as one with average levels of both. **Geometric** aggregation penalises imbalance, and non-compensatory methods go further (Greco et al., 2019). The right choice depends on whether the concept being measured allows such trade-offs.

Robustness

Because each step involves a judgement, a composite score carries uncertainty beyond sampling error. Uncertainty and sensitivity analysis re-computes the index under alternative normalisations, weights and aggregation rules and shows how much each unit's score or rank moves (Saisana et al., 2005). If a unit ranks fifth under one reasonable set of choices and fifteenth under another, its rank is not a precise fact.

Practical guidance for survey and business data

- **State the base period and the formula.** "Up 12%" means little without knowing the reference point and whether weights are fixed or chained.
- **Use weights that reflect spending or importance.** For an internal price tracker, weight items by revenue share rather than treating every product equally.
- **Keep the components visible.** Report the sub-indicators alongside a composite score so that users can see what drives a change.
- **Check correlations before weighting.** Highly correlated survey items can double-count one idea.
- **Run a sensitivity check.** Even a simple comparison of equal versus alternative weights shows whether conclusions are robust.

Common mistakes

- **Comparing index levels across different bases.** An index of 120 on a 2015 base and 110 on a 2020 base cannot be compared directly without rebasing.
- **Treating a composite as a measurement.** A composite indicator is a model of a concept, not an observation of it, and it can be shaped to support a desired message (Saltelli, 2007).

Key takeaways

Index numbers and composite indicators turn many figures into one, which makes them powerful for communication and monitoring. Their results depend on the base period, the formula, the normalisation, the weights and the aggregation rule. Good practice is to make each of these choices explicit, prefer superlative or chained price indices when quantities shift, show the components of any composite score and test how sensitive the conclusions are to reasonable alternatives.

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